Exercise 4.1.11 Calculate x̄ and s by hand using the 2-pass algorithm, the 1-pass algorithm, and Welford’s algorithm in the following two cases. (a) The data based on n = 3 observations: x 1 = 1, x 2 = 6, and x 3 = 2. (b) The sample path x(t) = 3 for 0 < t ≤ 2, and x(t) = 8 for 2 < t ≤ 5, over the time interval 0 < t ≤ 5.

A-)

n =3 x1 = 1 x2 = 6 x3 = 2

**2 pass algorithm**

x¯ = 1 + 6 + 1 / 3 = 3

s = √(1-3)²+(6-3)²+(2-3)/3) = √(4+9+1/3) = √(4,667)

**1 pass algorithm**

√s² = √(((1²+6²+1² )/ 3) – ((1+6+2)/3)²)

√(41-27)/3 = √(14/3)

x¯ = √9 = 3

**Welford**

x¯1 = 0+1 = 1

x¯2 = 1+ (6-1)/2 = 3,5

x¯3 = 3,5 + (2-3,5)/3 = 3

V1 = 0

V2 = 0+(6-1)/2 = 5/2

V3 = 2,5 + 2 \* (2 – 3,5) /3

B-)

X(t) = 3 0< t < 2

8 2<2<=5

**Two pass**

X = 1/5 \* [ 3\* (2-0) + 8 \*(5 -2)] = 30/5 = 6

3² =[ integral de 0 a 2 (3-6) dt + integral de 2 a 5(8-6)² dt

= 1/5 \* (18+12) = 6

S² = 6

√6 = 2,44

**One pass**

S² = [1/5 (9\*(2-0) + 64 (5-2)

S² = 1/5 \* (18+192) – 6 = 210/5 – 6 = 42 -36

= √6 = 2,44

X = 6

**Welford**

X1 = 0 + 1 \* (3-0)/2 = 3

X2 = 3 + 3\*(8-3)/5 = 3+3 = 6

V1 = 0 + 0\*(3-0)² = 0

V2 = 0 + (3\*2) /5 \* (8-0)²

= 6.64/5 = √6